The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Autumn Semester 2010–11**

Differential and difference equations **2 hours**

Attempt all the questions. The allocation of marks is shown in brackets.

1 (i) Show that $y = \sqrt{x^4 + 2x}$ is a particular solution of the differential equation

$$y' = \frac{2y}{x} - \frac{3}{y}.$$

(6 marks)

(ii) Determine the solution of the differential equation

$$\frac{dy}{dx} = \frac{1}{x^2 + 7x + 12},$$

by the method of direct integration subject to the initial condition $y(0) = 0$.
(9 marks)

(iii) Given the initial condition $y(0) = -1$, find the particular solution to the differential equation

$$\frac{dy}{dx} = \frac{x\tan(x^2)}{2y}.$$

(10 marks)

2 (i) Use the integrating factor method to find the particular solution to the linear inhomogeneous differential equation

$$\frac{1}{2}\frac{dy}{dx} + \frac{y}{x} - \frac{1}{2}\sin x = 0,$$

subject to the condition $y(\pi) = 0$. ***(10 marks)***

(ii) Find the particular solution of the differential equation

$$y'' + 2y' - 3y = 5\sin(3x) + x + 1,$$

subject to the conditions $y(0) = 0$ and $y'(0) = -1$. ***(15 marks)***

3 (i) Let a, K, and N_0 be positive and real numbers such that $0 < N_0 < K$. Use the separation of variables to show that the solution of the logistic equation

$$\frac{dN}{dt} = aN\left(1 - \frac{N}{K}\right)$$

which satisfies the initial condition $N = N_0$ when $t = 0$ is

$$N = \frac{K}{\left(\frac{K}{N_0} - 1\right) e^{-at} + 1}$$

What happens with the population N when $t \to \infty$? ***(10 marks)***

(ii) Solve the difference equation

$$u_{n+2} - 2u_{n+1} - 3u_n = \cos\frac{n\pi}{2} + n^2 + 1$$

subject to the conditions $u_0 = 0$ and $u_1 = 1$. ***(15 marks)***

4 (i) The interaction between three species is described by the system

$$\frac{dx}{dt} = x(-2 + y - z), \quad \frac{dy}{dt} = y(1 - x - z), \quad \frac{dz}{dt} = z(3 + x - y),$$

with $x = x(t)$, $y = y(t)$, $z = z(t)$. Find all *five* equilibrium levels of this model. ***(6 marks)***

(ii) P_n denotes the number of binary operations which take place in part of a computer program when n data points are supplied. If P_n satisfies the equation

$$P_{n+2} - P_{n+1} = 2(P_{n+1} - P_n),$$

and the conditions $P_0 = 10$ and $P_3 = 24$, obtain an expression for P_n in terms of n. ***(5 marks)***

(iii) Obtain the solution for the Bernoulli-type differential equation

$$\frac{dy}{dx} + \frac{y}{x} = xy^2, \quad y = y(x),$$

subject to $y(1) = 1/2$. ***(14 marks)***

End of Question Paper