The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS — Spring Semester 2014–2015

Mathematical Methods for Statistics — 2 hours

RESTRICTED OPEN BOOK EXAMINATION
Candidates may bring to the examination lecture notes and associated lecture material (including set textbooks) plus a calculator that conforms to University regulations.
Candidates should attempt ALL questions.
The paper will be marked out of 80 and the allocation of marks is shown in brackets.

1 (i) Compute $\displaystyle\sum_{k=1}^{\infty} \frac{1}{3^k}$

(ii) Find constants A and B so that:

$$\frac{1}{4n^2-1} = \frac{A}{2n-1} + \frac{B}{2n+1}$$

(iii) Compute $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4n^2-1}$ ***(10 marks)***

2 Compute the derivatives of the following functions with respect to x.

(i) $r(x) = x^2 \ln(x)$

(ii) $s(x) = e^{x+x^{-1}}$

(iii) $t(x) = \dfrac{cos(x)}{sin(x)}$

(10 marks)

3 Find and classify all critical points of $f(x, y) = 6x^2 + 6xy + y^2 - 2y$ ***(10 marks)***

4 Compute the definite integrals

(i) $$\int_{-2}^{2} \frac{1}{2x+5} dx$$

(ii) $$\int_{0}^{\pi/4} x \sin(2x) dx$$

(10 marks)

5 Let D be the region bounded by the inequalities $x \geq 0, y \geq 0$, and $x^2 + y^2 \leq 1$. Find $\iint_D x \, dx \, dy$. ***(10 marks)***

6 Use Gaussian elimination to solve the following system of equations:

$$x + 2y + 3z = 6$$
$$2x - y + z = 0$$
$$3x - 2y - z = -2$$

(10 marks)

7 Let

$$M = \begin{pmatrix} 4 & 3 \\ 1 & 1 \end{pmatrix}, \quad v = \begin{pmatrix} 1 & -1 \end{pmatrix}, \quad w = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

Find:

(i) vM

(ii) Mw

(iii) M^{-1}

(10 marks)

8 Let

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

(i) Find the eigenvectors of A and the associated eigenvalues.

(ii) Use your result from Part (i) to find

$$A^{100} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

(10 marks)

End of Question Paper