The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2013–2014**

History of Mathematics **2 hours 30 minutes**

Answer Question 1 and three other questions. If you answer more than three of the Questions 2 to 5, only your best three will be counted.

1 Attempt *three* of questions **(a)**, **(b)**, **(c)**, **(d)** below. If you attempt more than three, only your best three will be counted

(a) Below is a translation of **Problem 5** on the Babylonian clay tablet **BM 13901**.

Problem: *I add the area and four thirds of the side of my square*: $0;55$. *What is the side*?
Solution: *Four thirds* $1;20$. *Halve* $0;40$. *Square* $0;26,40$. *Add on* $0;55$: $1;21,40$. *Square root*: $1;10$. *Subtract* $0;40$ (*that was squared*), $0;30$ *is the side.*

Write each of the *seven* different numbers occurring in the extract as a fraction of the form m/n, where m and n are positive integers. ***(2 marks)***

Explain the extract, commenting on aspects of Babylonian mathematics that it illustrates. How does the tablet **BM 13901** *itself* suggest a purpose for the problem? ***(5 marks)***

(b) What was the Pythagoreans' motto? To what *two* aspects of numbers, *not* in Euclid's *Elements*, but considered in (i) and (ii) below, did it lead them? ***(1 mark)***

(i) Given that $2^2 \times 71$ is *one* member of an amicable pair, *determine* the other as a product of primes. ***(3 marks)***

(ii) Let $m \geqslant 3$ and n be positive integers. Give the *first term* and the *common difference* of the arithmetic progression, the sum of whose first n terms is the nth m-gonal number. Deduce that the sum of the first n odd numbers is n^2. ***(3 marks)***

(c) Set the following extract into its *mathematical* context.

... almost one quarter of an hour was spent, each beholding the other with admiration before one word was spoke ... . ***(5 marks)***

What relevance does the *current* year 2014 have to the extract? ***(2 marks)***

(d) Describe the role played by the *cycloid* in developing the calculus. ***(3 marks)***

Name *four* seventeenth century mathematicians, of *differing* nationalities, who studied the curve, mentioning *one* contribution of each. ***(4 marks)***

2 Why could translation of the *Rhind Papyrus* begin soon after its arrival at the British Museum? Which Egyptologist *first* translated it and how did he leave his mark on all subsequent studies of the Papyrus? ***(4 marks)***

(a) What *rule* did the Egyptians use for finding the area of a circle? Find the *implicit* value of π to which this gives rise. Describe in some detail the *two* problems on the Rhind Papyrus that use this rule directly to find the area of a circle. ***(8 marks)***

(b) Problem 42 of the *Rhind Papyrus* asks for the volume of a cylindrical granary with *height* 10 and base *diameter* 10. The answer found there reduces to $a\,\overline{18}\ \overline{b}\ \overline{54}\ \overline{81}$, where a and b are positive integers. Determine a and b. ***(4 marks)***

3 **(a)** Set the following *two* quotations ascribed to Euclid into context. Indicate the questions to which he was responding and the points that he was making in his replies.

(i) *There is no royal road to geometry.*

(ii) *Give him a coin, for he must profit from what he learns.* ***(4 marks)***

(b) Over a thousand editions of Euclid's *Elements* have appeared and it has been translated into many languages. Where, when, and in what language did the *original* appear? What were the first *two* languages into which it was translated. Comment on the editions/translations produced in the years **1482**, **1543**, **1570**. ***(8 marks)***

(c) Comment on the following extract from John Aubrey's *Brief Lives* of how the philosopher Thomas Hobbes came to fall in love with geometry one day in 1628.

He was 40 years old before he looked on Geometry, which happened accidentally. Being in a Gentleman's Library, Euclid's Elements *lay open at the* 47 El. libri I. *He read the Proposition. By G—, sayd he (he would now and then sweare an Oath by way of emphasis) this is impossible! So he reads the Demonstration of it, which referred him back to such a Proposition, which proposition he read. That referred him back to another, which he also read. Et sic deinceps (and so on) that at last he was demonstratively convinced of that trueth. That made him fall in love with Geometry.* ***(4 marks)***

4 For what *varying* reasons did secrecy play a part in the work of Scipione del Ferro, Tartaglia and Cardan? ***(7 marks)***

Below is a translation of *six* lines of a verse confided to Cardan by Tartaglia giving a solution to a cubic equation of the form: *cube equals things and numbers.*

You divide the number into two parts,
So that one times the other produces
The cube of a third of the things.
Then of these two parts,
You take the cube roots added together,
And this will be your thought.

Given that $x^3 = 6x + 40$ has only *one* real root, use the above lines to show that it is

$$\sqrt[3]{20 + \sqrt{392}} + \sqrt[3]{20 - \sqrt{392}},$$

and, giving reasons, *simplify* this expression involving root extractions. ***(9 marks)***

5 Why is more known about the life and work of Archimedes than any other Greek mathematician? ***(4 marks)***

Use the following two propositions from **Book I** of his *On the Sphere and the Cylinder* to express the surface area S and the volume V of a sphere in terms of its radius r. [You may assume results for the area of a circle in terms of its radius, and the volume of a cone in terms of its height and base radius.]

Proposition 33 The surface area of a sphere is *four* times the greatest circle in it.

Proposition 34 The volume of a sphere is *four* times the cone which has as its base the greatest circle in the sphere and its height equal to the radius of the sphere. ***(2 marks)***

What, according to the Greek biographer Plutarch, did Archimedes request be engraved on his tombstone? State and verify the result underlying his request. ***(4 marks)***

In **Proposition 2** of his *Method*, Archimedes considers a cylinder with base radius $2r$, and axis the line segment from $(0, 0, 0)$ to $(2r, 0, 0)$. He shows that a sphere of radius r and a cone with both base radius and height equal to $2r$ will, when suspended together from their *centroids* at the point $(-2r, 0, 0)$, balance the cylinder *in situ* about a fulcrum at $(0, 0, 0)$. Use his result to express the volume of a sphere in terms of its radius r.

(6 marks)

End of Question Paper