The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2012–2013**

Algebraic Topology **2 hours 30 minutes**

Answer ***all four*** *questions. All questions carry equal weight. You will be marked on correctness/completeness and rigour/presentation. Justify all your answers.*

1 What does it mean for spaces X and Y to be homotopy equivalent? Which of the following spaces are homotopy equivalent? Justify your answers.

(i) The plane $\mathbb{R}^2$.

(ii) The punctured plane $\mathbb{R}^2 \setminus 0$.

(iii) The circle S^1.

(iv) A space of your choice.

2 Classify the connected covering spaces of the space $D^2 \vee \mathbb{R}P^2$.

3 Let X be the space obtained from the square $I \times I$ by identifying the corners. Find the fundamental group and all homology groups of X and check your answers.

4 (i) Explain briefly how a short exact sequence of chain complexes gives rise to a long exact sequence of homology groups.

(ii) For an exact sequence $A \xrightarrow{\alpha} B \xrightarrow{\beta} C \xrightarrow{\gamma} D \xrightarrow{\delta} E$ of Abelian groups, show that $C = 0$ iff the map $A \xrightarrow{\alpha} B$ is surjective and the map $D \xrightarrow{\delta} E$ is injective. Hence for a subspace A of X, show that the inclusion $A \hookrightarrow X$ induces isomorphisms on all homology groups iff $H_n(X, A) = 0$ for all n.

(iii) Give a counterexample to show that $H_n(X, A)$ is not necessarily isomorphic to $H_n(X/A)$.

End of Question Paper